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ARTICLE

Effect of rotation on plane waves in generalized thermomicrostretch elastic solid: comparison of different theories using finite element method

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Abstract: The present investigation is aimed at studying the effect of rotation on the thermomicrostretch elastic solid. The formulation is applied in the context of the generalized thermoelasticity Lord–Şhulman theory with one relaxation time and Green–Lindsay's theory with two relaxation times, as well as the classical dynamical coupled theory. The problem has been solved numerically using a finite element method. Numerical results for the temperature distribution, the displacement components, the force stresses, the couple stresses, and the microstress distribution are represented graphically. The results indicate that the effects of rotation are very pronounced. Comparisons are made with the results in the presence and absence of rotation and in the presence and absence of microstretch constants between the two theories.

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Résumé : La présente vise l'étude des effets de la rotation sur le micro-étirement thermique de solides élastiques. Nous travaillons dans le contexte de la théorie de la thermo-élasticité de Lord–Shulman avec un temps de relaxation et de la théorie de Green–Lindsay avec deux temps de relaxation, ainsi que de la théorie de la dynamique classique couplée. Nous solutionnons numériquement par méthode d'éléments finis. Nous présentons graphiquement les résultats numériques pour la distribution de température, les composants de déplacement, les forces de charge, les couples de charge et la distribution de micro-efforts. Les résultats indiquent que les effets de la rotation sont très prononcés. Nous comparons les résultats pour les deux théories, avec et sans rotation, ainsi qu'en présence et absence de constante de micro-étirement. [Traduit par la Rédaction]

1. Introduction

It is well known that material response to external stimuli depends heavily on the motion of its inner structures. Classical elasticity ignores this effect by ascribing only translational degrees of freedom to material points of the body. The micropolar theory of Eringen [1], by including intrinsic rotations of the microstructure, provides a model that can support the body and surface couples and display the high-frequency optical branch of the wave spectrum. For engineering applications, it can model composites with rigid chopped fibers, elastic solids with rigid granular inclusions, and other industrial materials, such as liquid crystals [2–4]. The next step, in the enhancement of the domain of applications of the theory, is to include microstructural expansions and contractions. The linear theory of micropolar elasticity is adequate to represent the behavior of such materials. For ultrasonic waves (i.e., in the case of elastic vibrations characterized by high frequencies and small wavelengths), the influence of the body microstructure becomes significant. This influence of microstructure results in the development of new types of waves that are not in the classical theory of elasticity. Metals, polymers, composites, solids, rocks, and concrete are typical media with microstructures. More generally, most natural and manmade materials, including engineering, geological, and biological media, possess a microstructure. Agarwal [5, 6] studied thermoelastic and magnetothermoelastic plane wave propagation in an infinite nonrotating medium. Some problems in thermoelastic rotating media are due to Schoenberg and Censor [7], Puri [8], Roy Choudhuri and Debnath [9, 10], and Othman [11, 12]. The propagation of plane harmonic waves in a rotating elastic medium without thermal field has been studied. It was shown there that rotation causes the elastic medium to be dispersive and anisotropic. These problems are based on the more realistic elastic model because earth, the moon, and other plants have angular velocity.

Eringen and Şuhubi [13] and Eringen [1] developed the linear theory of micropolar elasticity. Othman [14] studied the relaxation effects of thermal shock problems in elastic half space of generalized magnetothermoelastic waves under three theories. Eringen [15] introduced the theory of microstretch elastic solids. This theory is a generalization of the theory of micropolar elasticity [16] and a special case of the micromorphic theory. The material points of microstretch elastic solids can stretch and contract independently of their translations and rotations. The microstretch continua are used to characterize composite materials and various porous media [17]. The basic results in the theory of microstretch elastic solids were obtained in the literature [18–21]. The development of the effect of rotation and magnetic field are available in many studies, such as refs. 22–27.

The theory of thermomicrostretch elastic solids was introduced by Eringen [16]. In the framework of the theory of thermomicrostretch solids Eringen established a uniqueness theorem for the mixed initial boundary value problem. The theory was illustrated through the solution of one-dimensional wave and compared with lattice dynamical results. The asymptotic behavior of solutions and an existence result were presented by Bofill and

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Quintanilla [28]. A reciprocal theorem and a representation of Galerkin type were presented by De Cicco and Nappa [29]. De Cicco and Nappa [30] extended a linear theory of thermomicrostretch elastic solids that permits the transmission of heat as thermal waves at finite speed. The theory is based on the entropy production inequality proposed by Green and Laws [31]. De Cicco and Nappa [29] studied the uniqueness of the solution of the mixed initial boundary value problem is also investigated. The basic results and an extensive review of the theory of thermomicrostretch elastic solids can be found in Eringen [18].

The coupled theory of thermoelasticity has been extended by including the thermal relaxation time in the constitutive equations by Lord and Shulman (L-S) [32] and Green and Lindsay (G-L) [33]. These theories eliminate the paradox of infinite velocity of heat propagation and are termed generalized theories of thermoelasticity. Othman et al. [34] studied the fundamental solution on plane waves in a generalized thermomicrostretch elastic half space under three theories. The finite element method is a powerful technique originally developed for numerical solution of complex problems in structural mechanics, and it remains the method of choice for complex systems. A further benefit of this method is that it allows physical effects to be visualized and quantified regardless of experimental limitations. Abbas and co-workers [35–38] have successfully applied the finite element method to various problems in generalized thermoelastic materials.

The purpose of the present paper is to obtain the normal displacement, temperature, normal force stress, and tangential couple stress in a microstretch elastic solid under the effect of rotation. A finite element method is used to obtain the expressions of the considered variables. The distributions of the considered variables are represented graphically. A comparison is carried out between the temperature, stresses, and displacements as calculated from the generalized thermoelasticity L-S, G-L, and classical dynamical coupled (CD) theories for the propagation of waves in a semi-infinite microstretch elastic solid.

2. Formulation of the problem

Following Eringen [7], L-S [32], and G-L [33], the constitutive equations and field equations for a linear isotropic generalized thermomicrostretch elastic solid in the absence of body forces are obtained. We consider Cartesian coordinate system (x, y, z) having originated on the surface $y = 0$ and z-axis pointing vertically into the medium. The thermoelastic plate is rotating uniformly with an angular velocity $\Omega = \Omega \boldsymbol{n}$, where $\boldsymbol{n}$ is a unit vector representing the direction of the axis of rotation. The basic governing equations of linear generalized thermoelasticity with rotation in the absence of body forces and heat sources are [39]

$$(\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}) + (\mu + k)\nabla^2 \boldsymbol{u} + k(\nabla \times \boldsymbol{\varphi}) + \lambda_0 \nabla \varphi^* - \hat{\gamma}\left(1 + \nu_0 \frac{\partial}{\partial t}\right)\nabla T = \rho[\ddot{\boldsymbol{u}} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \boldsymbol{u}) + 2\Omega \times \dot{\boldsymbol{u}}] \tag{1}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot \boldsymbol{\varphi}) - \gamma \nabla \times (\nabla \times \boldsymbol{\varphi}) + k(\nabla \times \boldsymbol{u}) - 2k\boldsymbol{\varphi} = j\rho \frac{\partial^2 \boldsymbol{\varphi}}{\partial t^2} \tag{2}$$

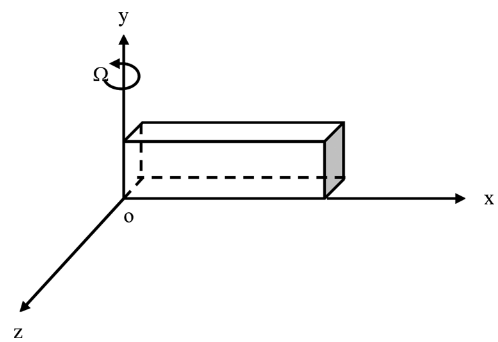
$$\alpha_0 \nabla^2 \varphi^* - \frac{1}{3}\lambda_1 \varphi^* - \frac{1}{3}\lambda_0(\nabla \cdot \boldsymbol{u}) + \frac{1}{3}\hat{\gamma}_1\left(1 + \nu_0 \frac{\partial}{\partial t}\right)T = \frac{3}{2}\rho j \frac{\partial^2 \varphi^*}{\partial t^2} \tag{3}$$

$$K\nabla^2 T = \rho C_E\left(n_1 + \tau_0 \frac{\partial}{\partial t}\right)\dot{T} + \hat{\gamma} T_0\left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\dot{e} + \hat{\gamma}_1 T_0\left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial \varphi^*}{\partial t} \tag{4}$$

$$\sigma_{il} = \left(\lambda_0 \phi^* + \lambda u_{r,r}\right)\delta_{il} + (\mu + k)u_{l,i} + \mu u_{i,l} - k\varepsilon_{ilr}\varphi_r - \hat{\gamma}\left(1 + \nu_0 \frac{\partial}{\partial t}\right)T\delta_{il} \tag{5}$$

$$m_{il} = \alpha \phi_{r,r}\delta_{il} + \beta \phi_{i,l} + \gamma \phi_{l,i} + b_0 \varepsilon_{ilr}\phi^*_{,r} \tag{6}$$

Geometry of the problem



$$\lambda_i = \alpha_0 \phi^*_{,i} + b_0 \varepsilon_{ilr}\phi_{l,r} \tag{7}$$

$$e = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \tag{8}$$

The state of plane strain parallel to the xz-plane is defined by

$$u_1 = u(x, z, t) \quad u_2 = 0 \quad u_3 = w(x, z, t) \quad \varphi_1 = \varphi_3 = 0$$
$$\varphi_2 = \varphi_2(x, z, t) \quad \varphi^* = \varphi^*(x, z, t) \quad \Omega = (0, \Omega, 0) \tag{9}$$

Field equations (1)–(4) reduce to

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) + (\mu + k)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - k\frac{\partial \phi_2}{\partial z} + \lambda_0 \frac{\partial \varphi^*}{\partial x} - \gamma\left(1 + \nu_0 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \rho\left[\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega \frac{\partial w}{\partial t}\right] \tag{10}$$

$$(\lambda + \mu)\left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right) + (\mu + k)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + k\frac{\partial \phi_2}{\partial x} + \lambda_0 \frac{\partial \varphi^*}{\partial z} - \hat{\gamma}\left(1 + \nu_0 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \rho\left[\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega \frac{\partial u}{\partial t}\right] \tag{11}$$

$$\gamma\left(\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2}\right) - 2k\phi_2 + k\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) = j\rho \frac{\partial^2 \phi_2}{\partial t^2} \tag{12}$$

$$\alpha_0\left(\frac{\partial^2 \varphi^*}{\partial x^2} + \frac{\partial^2 \varphi^*}{\partial z^2}\right) - \frac{1}{3}\lambda_1 \varphi^* - \frac{1}{3}\lambda_0\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + \frac{1}{3}\hat{\gamma}_1\left(1 + \nu_0 \frac{\partial}{\partial t}\right)T = \frac{3}{2}\rho j \frac{\partial^2 \varphi^*}{\partial t^2} \tag{13}$$

$$\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho C_E\left(n_1 + \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \hat{\gamma} T_0\left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial e}{\partial t} + \hat{\gamma}_1 T_0\left(n_1 + n_0 \tau_0 \frac{\partial}{\partial t}\right)\frac{\partial \varphi^*}{\partial t} \tag{14}$$

where

$$\hat{\gamma} = (3\lambda + 2\mu + k)\alpha_{t_1} \qquad \hat{\gamma}_1 = (3\lambda + 2\mu + k)\alpha_{t_2} \quad \text{and} \quad \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} \tag{15}$$

The constants $\hat{\gamma}$ and $\hat{\gamma}_1$ depend on mechanical as well as thermal properties of the body and the dots denote partial derivative with respect to time.

Equations (10)–(14) are the field equations of the generalized thermomicrostretch elastic solid, applicable to the L-S theory, and the CD theory, as follows:

2.1. CD theory

The equations of the CD thermo-microstretch theory, when

$$n_0 = 0 \qquad n_1 = 1 \qquad \tau_0 = 0 \qquad \nu_0 = 0 \tag{16}$$

Equations (10), (11), (13), and (14) have the form

$$\rho\left(\frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t}\right) = (\lambda + \mu)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 w}{\partial x \partial z}\right) + (\mu + k)\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - k\frac{\partial \phi_2}{\partial z} + \lambda_0\frac{\partial \varphi^*}{\partial x} - \hat{\gamma}\frac{\partial T}{\partial x} \tag{17}$$

$$\rho\left(\frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t}\right) = (\lambda + \mu)\left(\frac{\partial^2 u}{\partial x \partial z} + \frac{\partial^2 w}{\partial z^2}\right) + (\mu + k)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right) + k\frac{\partial \phi_2}{\partial x} + \lambda_0\frac{\partial \varphi^*}{\partial z} - \hat{\gamma}\frac{\partial T}{\partial z} \tag{18}$$

$$c_3^2\left(\frac{\partial^2 \varphi^*}{\partial x^2} + \frac{\partial^2 \varphi^*}{\partial z^2}\right) - c_4^2\varphi^* - c_5^2\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right) + c_6^2 T = \frac{\partial^2 \varphi^*}{\partial t^2} \tag{19}$$

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho C_E\frac{\partial T}{\partial t} + \hat{\gamma}T_0\frac{\partial e}{\partial t} + \hat{\gamma}_1 T_0\frac{\partial \varphi^*}{\partial t} \tag{20}$$

The constitutive relation can be written as

$$\sigma_{xx} = \lambda_0\phi^* + (\lambda + 2\mu + k)\frac{\partial u}{\partial x} + \lambda\frac{\partial w}{\partial z} - \hat{\gamma}T \tag{21}$$

$$\sigma_{zz} = \lambda_0\phi^* + (\lambda + 2\mu + k)\frac{\partial w}{\partial z} + \lambda\frac{\partial u}{\partial x} - \hat{\gamma}T \tag{22}$$

$$\sigma_{xz} = \mu\frac{\partial u}{\partial z} + (\mu + k)\frac{\partial w}{\partial x} + k\varphi_2 \tag{23}$$

$$\sigma_{zx} = \mu\frac{\partial w}{\partial x} + (\mu + k)\frac{\partial u}{\partial z} + k\varphi_2 \tag{24}$$

$$m_{xy} = \gamma\frac{\partial \phi_2}{\partial x} + b_o\frac{\partial \phi^*}{\partial z} \tag{25}$$

$$m_{zy} = \gamma\frac{\partial \phi_2}{\partial z} - b_o\frac{\partial \phi^*}{\partial x} \tag{26}$$

$$\lambda_x = \alpha_o\frac{\partial \phi^*}{\partial x} + b_o\frac{\partial \phi_2}{\partial z} \tag{27}$$

$$\lambda_z = \alpha_o\frac{\partial \phi^*}{\partial z} - b_o\frac{\partial \phi_2}{\partial x} \tag{28}$$

where

$$c_3^2 = \frac{2\alpha_0}{3\rho j} \qquad c_4^2 = \frac{2\lambda_1}{9\rho j} \qquad c_5^2 = \frac{2\lambda_0}{9\rho j} \qquad c_6^2 = \frac{2\hat{\gamma}_1}{9\rho j} \tag{29}$$

2.2. L-S theory

When

$$n_1 = n_0 = 1 \qquad \tau_0 > 0 \qquad \nu_0 = 0 \tag{30}$$

Equations (10), (11), and (13) are the same as (17), (18), and (19), and (14) has the form

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)(\rho C_E T + \hat{\gamma}T_0 e) + \hat{\gamma}_1 T_0\frac{\partial \varphi^*}{\partial t} \tag{31}$$

2.3. G-L theory

When

$$n_1 = 1 \qquad n_0 = 0 \qquad \nu_0 \geq \tau_0 > 0 \tag{32}$$

Equations (10), (11), and (13) remain unchanged and (14) has the form

$$K\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \rho C_E\left(1 + \tau_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial t} + \hat{\gamma}T_0\frac{\partial e}{\partial t} + \hat{\gamma}_1 T_0\frac{\partial \varphi^*}{\partial t} \tag{33}$$

2.4. Without stretch

The corresponding equations for the generalized micropolar thermoelasticity without stretch can be obtained from the preceding cases by taking

$$\alpha_0 = \lambda_0 = \lambda_1 = \phi^* = 0 \tag{34}$$

For convenience, the following nondimensional variables are used

$$\begin{gathered}\bar{x}_i = \frac{\omega^*}{c_2}x_i \qquad \bar{u}_i = \frac{\rho c_2\omega^*}{\hat{\gamma}T_0}u_i \qquad \bar{t} = \omega^* t \qquad \bar{\tau}_0 = \omega^*\tau_0 \\ \bar{\nu}_0 = \omega^*\nu_0 \qquad \bar{T} = \frac{T}{T_0} \qquad \bar{\sigma}_{ij} = \frac{\sigma_{ij}}{\hat{\gamma}T_0} \qquad \bar{m}_{ij} = \frac{\omega^*}{c_2\hat{\gamma}T_0}m_{ij} \\ \bar{\varphi}_2 = \frac{\rho c_2^2}{\hat{\gamma}T_0}\varphi_2 \qquad \bar{\lambda}_3 = \frac{\omega^*}{c_2\hat{\gamma}T_0}\lambda_3 \qquad \bar{\varphi}^* = \frac{\rho c_2^2}{\hat{\gamma}T_0}\varphi^* \\ \omega^* = \frac{\rho C_E c_2^2}{K} \qquad \bar{\Omega} = \frac{\Omega}{\omega^*} \qquad c_2^2 = \frac{\mu}{\rho}\end{gathered} \tag{35}$$

Using (35), (10)–(14) become (dropping the dashes for convenience)

$$\frac{\partial^2 u}{\partial x^2} + (a_1 + a_2)\frac{\partial^2 w}{\partial x \partial z} + (a_2 + a_3)\frac{\partial^2 u}{\partial z^2} + a_0\frac{\partial \phi^*}{\partial x} - a_3\frac{\partial \phi_2}{\partial z} - \left(1 + \nu_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2} - \Omega^2 u + 2\Omega\frac{\partial w}{\partial t} \tag{36}$$

$$\frac{\partial^2 w}{\partial z^2} + (a_1 + a_2)\frac{\partial^2 u}{\partial x \partial z} + (a_2 + a_3)\frac{\partial^2 w}{\partial x^2} + a_0\frac{\partial \phi^*}{\partial z} + a_3\frac{\partial \phi_2}{\partial x} - \left(1 + \nu_0\frac{\partial}{\partial t}\right)\frac{\partial T}{\partial z} = \frac{\partial^2 w}{\partial t^2} - \Omega^2 w - 2\Omega\frac{\partial u}{\partial t} \tag{37}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} = \left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T + \left(\frac{\partial}{\partial t} + n_0\tau_0\frac{\partial^2}{\partial t^2}\right) \times \left[\varepsilon_1\phi^* + \varepsilon_2\left(\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right)\right] \tag{38}$$

$$\frac{\partial^2 \phi_2}{\partial x^2} + \frac{\partial^2 \phi_2}{\partial z^2} + \zeta_1\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) - 2\zeta_1\phi_2 = \zeta_2\frac{\partial^2 \phi_2}{\partial t^2} \tag{39}$$